DBE

MATHEMATICS

PAST PAPER BOOKLET 2020

Past Paper Questions
By Topic

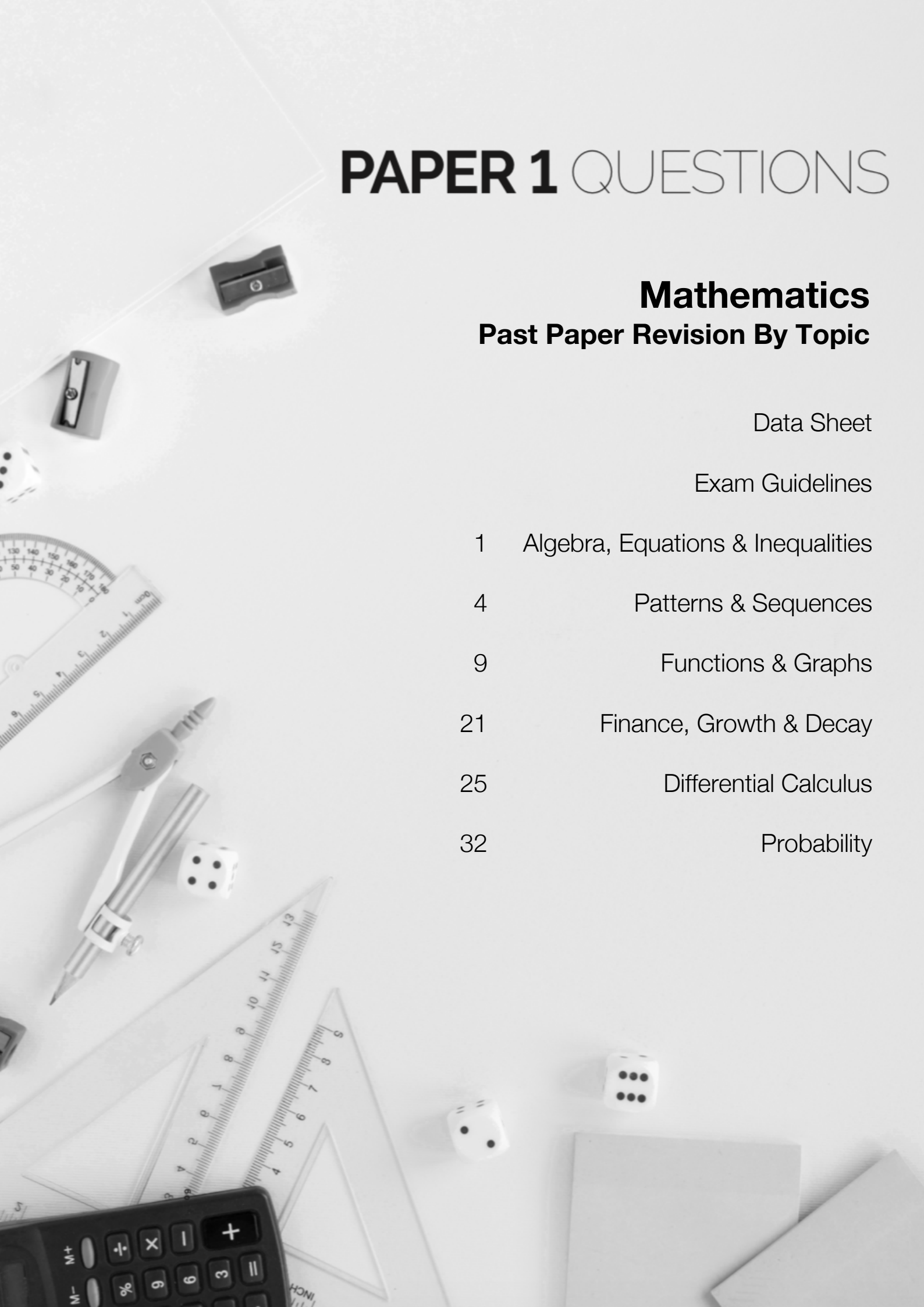
PAPER 1

AngloAmerican PLATINUM

XLEducation

PAPER 1 QUESTIONS

Mathematics
Past Paper Revision By Topic



5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC:\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

EXAMINATION GUIDELINES

SENIOR CERTIFICATE (SC)

GRADE 12

2015

These guidelines consist of 16 pages.

CONTENTS **Page**

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1. INTRODUCTION

The Curriculum and Assessment Policy Statement (CAPS) for Mathematics outlines the nature and purpose of the subject Mathematics. This guides the philosophy underlying the teaching and assessment of the subject in Grade 12.

The purpose of these Examination Guidelines is to provide clarity on the depth and scope of the content to be assessed in the Grade 12 Senior Certificate (SC) Examination in Mathematics.

These Examination Guidelines should be read in conjunction with:

- A resumé of subjects for the Senior Certificate
- Curriculum and Assessment Policy Statements for all approved subjects

2. ASSESSMENT IN GRADE 12

All candidates will write two question papers as prescribed.

2.1 Format of question papers for Grade 12

Paper	Topics	Duration	Total
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations and inequalities Differential Calculus Probability	3 hours	150
2	Euclidean Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150

Questions in both Papers 1 and 2 will assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of contexts.

An Information Sheet is included on p. 15.

2.2 Weighting of cognitive levels

Papers 1 and 2 will include questions across four cognitive levels. The distribution of cognitive levels in the papers is given below.

Cognitive level	Description of skills to be demonstrated	Weighting	Approximate number of marks in a 150-mark paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding • Candidates are expected to solve problems by integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks

3. ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as candidates should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process candidates must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $af(x)$ and $x=f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in Paper 2.

NUMBER PATTERNS, SEQUENCES AND SERIES

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FINANCE, GROWTH AND DECAY

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will not be examined in the examination.

ALGEBRA

1. Solving quadratic equations by completing the square will not be examined.
2. Solving quadratic equations using the substitution method (k-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

DIFFERENTIAL CALCULUS

1. The following notations for differentiation can be used: $f'(x)$, D_x, $\frac{dy}{dx}$ or y'.
2. In respect of cubic functions, candidates are expected to be able to:
   - Determine the equation of a cubic function from a given graph.

- Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
- Apply knowledge of transformations on a given function to obtain its image.

3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will not be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

PROBABILITY

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced is examinable.
3. Questions that require the candidate to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
   - The opposite angles of a cyclic quadrilateral are supplementary;
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
   - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
   - Equiangular triangles are similar.
4. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circle
   - Equal chords subtend equal angles at the circumference
   - Equal chords subtend equal angles at the centre
   - In equal circles, equal chords subtend equal angles at the circumference
   - In equal circles, equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
   - Tangents drawn from a common point outside the circle are equal in length.

5. The theory of quadrilaterals will be integrated into questions in the examination.
6. Concurrency theory is excluded.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.
   In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR ; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers.

4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}\odot$
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

7. CONCLUSION

This Examination Guidelines document is meant to articulate the assessment aspirations espoused in the CAPS document. It is therefore not a substitute for the CAPS document which educators should teach to.

Qualitative curriculum coverage as enunciated in the CAPS cannot be over-emphasised.

Question 1 — November 2014

1.1 Solve for x:

1.1.1 $(x-2)(4+x)=0$ (2)

1.1.2 $3x^2-2x=14$ (correct to TWO decimal places) (4)

1.1.3 $2^{x+2}+2^x=20$ (3)

1.2 Solve the following equations simultaneously:

$x=2y+3$

$3x^2-5xy=24+16y$ (6)

1.3 Solve for x: $(x-1)(x-2)<6$ (4)

1.4 The roots of a quadratic equation are: $x=\dfrac{3\pm\sqrt{-k-4}}{2}$

For which values of k are the roots real? (2)

[21]

Question 1 — Feb March 2015

1.1 Solve for x:

1.1.1 $x^2-x-20=0$ (2)

1.1.2 $2x^2-11x+7=0$ (correct to TWO decimal places) (3)

1.1.3 $5x^2+4>21x$ (5)

1.1.4 $2^{2x}-6.2^x=16$ (4)

1.2 Solve for x and y simultaneously:

$y+1=2x$

$x^2-xy+y^2=7$ (6)

1.3 The roots of a quadratic equation are given by $x=\dfrac{-5\pm\sqrt{20+8k}}{6}$, where $k\in\{-3;-2;-1;0;1;2;3\}$.

1.3.1 Write down TWO values of k for which the roots will be rational. (2)

1.3.2 Write down ONE value of k for which the roots will be non-real. (1)

1.4 Calculate a and b if $\sqrt{\dfrac{7^{2014}-7^{2012}}{12}}=a\left(7^b\right)$ and a is not a multiple of 7. (4)

[27]

Question 1 **November 2015**

1.1 Solve for x:

1.1.1 $x^2-9x+20=0$ (3)

1.1.2 $3x^2+5x=4$ (correct to TWO decimal places) (4)

1.1.3 $2x^{\frac{-5}{3}}=64$ (4)

1.1.4 $\sqrt{2-x}=x-2$ (4)

1.1.5 $x^2+7x<0$ (3)

1.2 Given: $(3x-y)^2+(x-5)^2=0$

Solve for x and y. (4)

1.3 For which value of k will the equation $x^2+x=k$ have no real roots? (4)

[26]

Question 1 **Feb March 2016**

1.1 Solve for x:

1.1.1 $x^2-x-12=0$ (3)

1.1.2 $x(x+3)-1=0$ (Leave your answer in simplest surd form.) (3)

1.1.3 $x(4-x)<0$ (3)

1.1.4 $x=\dfrac{a^2+a-2}{a-1}$ if $a=888\ 888\ 888\ 888$ (2)

1.2 Solve the following equations simultaneously:

$y+7=2x$ and $x^2-xy+3y^2=15$ (6)

1.3 Determine the range of the function $y=x+\dfrac{1}{x}$, $x\neq 0$ and x is real. (6)

[23]

Question 1 **May June 2016**

1.1 Solve for x:

1.1.1 $4x^2 - 25 = 0$ (3)

1.1.2 $x^2 - 5x - 2 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(2-x)(x+4) \geq 0$ (3)

1.1.4 $x - 3x^{\frac{1}{2}} = 4$ (5)

1.2 Solve for x and y:

$2x - y + 1 = 0$ and $x^2 - 3x - 4 - y = y^2$ (6)

1.3 Given: $f(x) = \sqrt{2x+1}$

1.3.1 Write down the domain of f. (1)

1.3.2 Solve for x if $f(x) = 2x - 1$. (5)

[26]

Question 1 **November 2016**

1.1 Solve for x:

1.1.1 $x(x-7) = 0$ (2)

1.1.2 $x^2 - 6x + 2 = 0$ (correct to TWO decimal places) (3)

1.1.3 $\sqrt{x-1} + 1 = x$ (5)

1.1.4 $3^{x+3} - 3^{x+2} = 486$ (4)

1.2 Given: $f(x) = x^2 + 3x - 4$

1.2.1 Solve for x if $f(x) = 0$ (2)

1.2.2 Solve for x if $f(x) < 0$ (2)

1.2.3 Determine the values of x for which $f'(x) \geq 0$ (2)

1.3 Solve for x and y: $x = 2y$ and $x^2 - 5xy = -24$ (4)

[24]

Question 2 **November 2014**

Given the arithmetic series: $2 + 9 + 16 + \ldots$ (to 251 terms).

2.1 Write down the fourth term of the series. (1)

2.2 Calculate the 251^{st} term of the series. (3)

2.3 Express the series in sigma notation. (2)

2.4 Calculate the sum of the series. (2)

2.5 How many terms in the series are divisible by 4? (4)
[12]

Question 3 **November 2014**

3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \ldots$

3.1.1 Write down the value of p. (2)

3.1.2 Determine the n^{th} term of the sequence. (4)

3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)

3.2 The first three terms of a geometric sequence are: 16 ; 4 ; 1

3.2.1 Calculate the value of the 12^{th} term. (Leave your answer in simplified exponential form.) (3)

3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)

3.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\ldots$ up to 98 factors. (4)
[19]

Question 2 **Feb March 2015**

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$T_2 - T_1 = 7$$
$$T_3 - T_2 = 13$$
$$T_4 - T_3 = 19$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)

[17]

Question 3 **Feb March 2015**

Consider the infinite geometric series: $45 + 40{,}5 + 36{,}45 + \ldots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_\infty - S_n < 1$? (5)

[11]

Question 2 **November 2015**

The following geometric sequence is given: $10 ; 5 ; 2{,}5 ; 1{,}25 ; \ldots$

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence. (2)

2.2 Determine the n^{th} term, T_n, in terms of n. (2)

2.3 Explain why the infinite series $10 + 5 + 2{,}5 + 1{,}25 + \ldots$ converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4)

[10]

Question 3 **November 2015**

Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the value of Q_{129}. (3)

[12]

Question 2 **Feb March 2016**

2.1 Given the following quadratic sequence: $-2 ; 0 ; 3 ; 7 ; ...$

2.1.1 Write down the value of the next term of this sequence. (1)

2.1.2 Determine an expression for the n^{th} term of this sequence. (5)

2.1.3 Which term of the sequence will be equal to 322? (4)

2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.

2.2.1 Determine the common difference of this sequence. (3)

2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)

2.2.3 Determine the sum of the first 50 terms of this sequence. (3)

[18]

Question 3 **Feb March 2016**

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	**During the second year**	**During the third year**
100 mm	70 mm	49 mm

3.1 Chris noted that the sequence of height increases, namely 100 ; 70 ; 49 ..., was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)

3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$. (3)

3.3 What height will the tree eventually reach? (3)

[10]

Question 2 **May June 2016**

2.1 Given the arithmetic series: $a + 13 + b + 27 + \ldots$

2.1.1 Show that $a = 6$ and $b = 20$ (2)

2.1.2 Calculate the sum of the first 20 terms of the series. (3)

2.1.3 Write the series in QUESTION 2.1.2 in sigma notation. (2)

2.2 Given the geometric series: $(x-2)+(x^2-4)+(x^3+2x^2-4x-8)+\ldots$

2.2.1 Determine the values of x for which the series converges. (4)

2.2.2 If $x = -\frac{3}{2}$, calculate the sum to infinity of the given series. (3)

[14]

Question 3 **May June 2016**

The first four terms of a quadratic number pattern are -1 ; 2 ; 9 ; 20.

3.1 Determine the general term of the quadratic number pattern. (4)

3.2 Calculate the value of the 48^{th} term of the quadratic number pattern. (2)

3.3 Show that the sum of the first differences of this quadratic number pattern can be given by $S_n = 2n^2 + n$ (3)

3.4 If the sum of the first 69 first differences in QUESTION 3.3 equals 9 591 (that is, $S_{69} = 9\,591$), which term of the quadratic number pattern has a value of 9 590? (2)

[11]

Question 2 **November 2016**

Given the finite arithmetic sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87

2.1 Write down the fourth term (T_4) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: 5 ; 1 ; –3 ; ... ; –83 ; –87 ; ... ; –4 187
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)

[11]

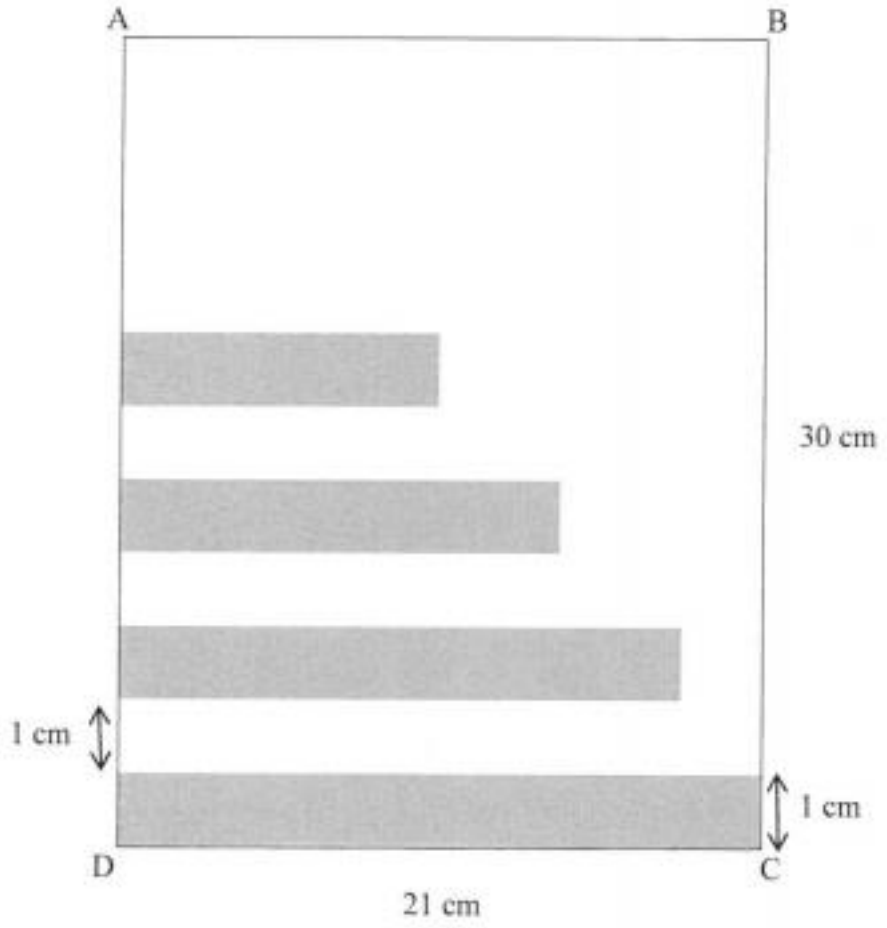
Question 3 **November 2016**

3.1 The first four terms of a quadratic number pattern are $-1\ ;\ x\ ;\ 3\ ;\ x+8$

3.1.1 Calculate the value(s) of x. (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.

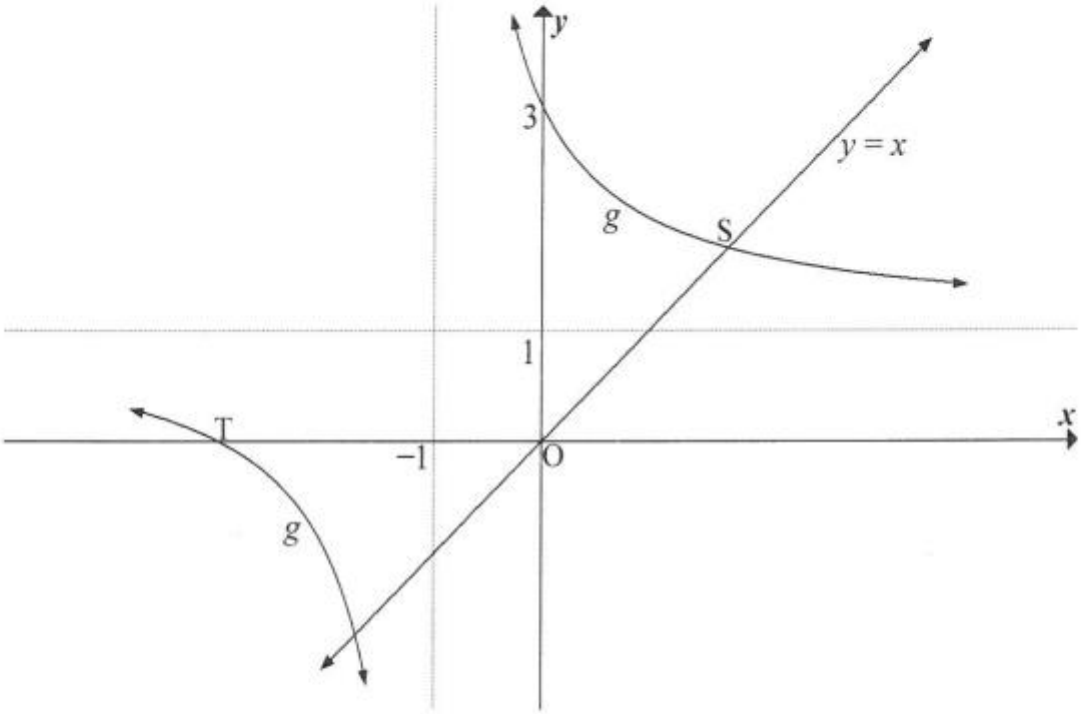


3.2.1 Calculate the length of the 10^{th} rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)
[15]

Question 4 November 2014

The diagram below shows the hyperbola g defined by $g(x)=\frac{2}{x+p}+q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at $(0 ; 3)$. The line $y=x$ intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of p and q. (2)

4.2 Calculate the x-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x)=g(x+5)$ (1)

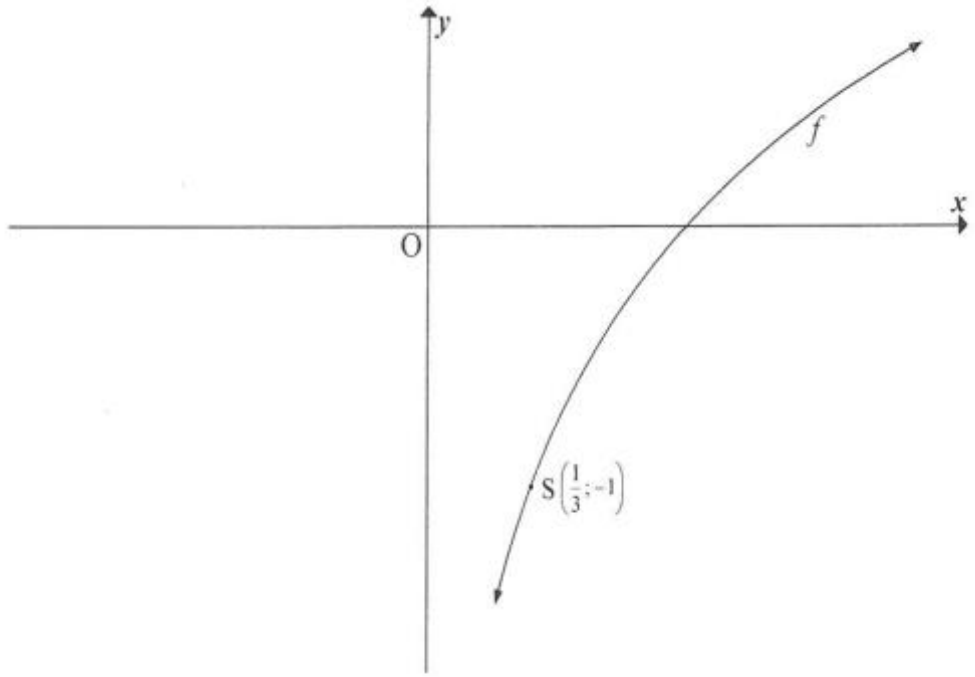
4.4 Calculate the length of OS. (5)

4.5 For which values of k will the equation $g(x)=x+k$ have two real roots that are of opposite signs? (1)

[11]

Question 5 November 2014

Given: $f(x)=\log_a x$ where $a>0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of f.



5.1 Prove that $a = 3$. (2)

5.2 Write down the equation of h, the inverse of f, in the form $y = ...$ (2)

5.3 If $g(x) = -f(x)$, determine the equation of g. (1)

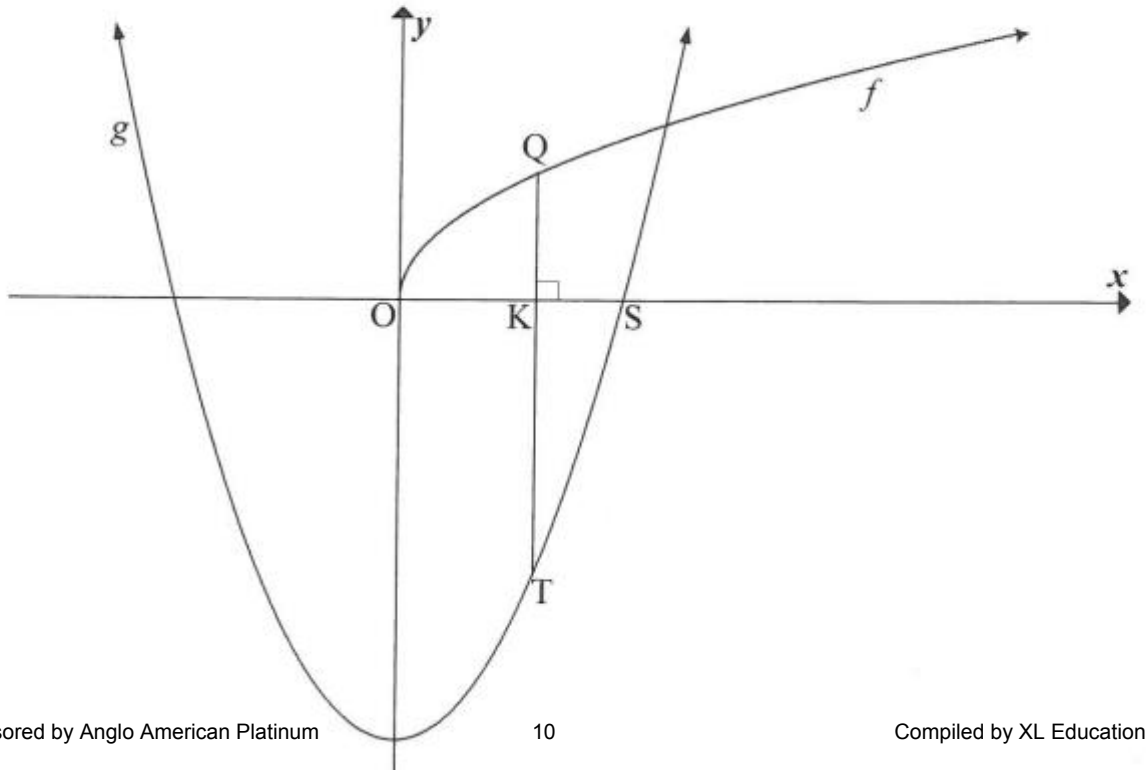
5.4 Write down the domain of g. (1)

5.5 Determine the values of x for which $f(x) \geq -3$. (3)

[9]

Question 6 **November 2014**

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



6.1 Determine the x-coordinate of S, correct to TWO decimal places. (2)

6.2 Write down the coordinates of the turning point of g. (2)

6.3 6.3.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K. (3)

6.3.2 Calculate the maximum length of QT. (6)
[13]

Question 4 **Feb March 2015**

Given: $g(x) = \frac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y-intercept of g (1)

4.2.2 The x-intercept of g (2)

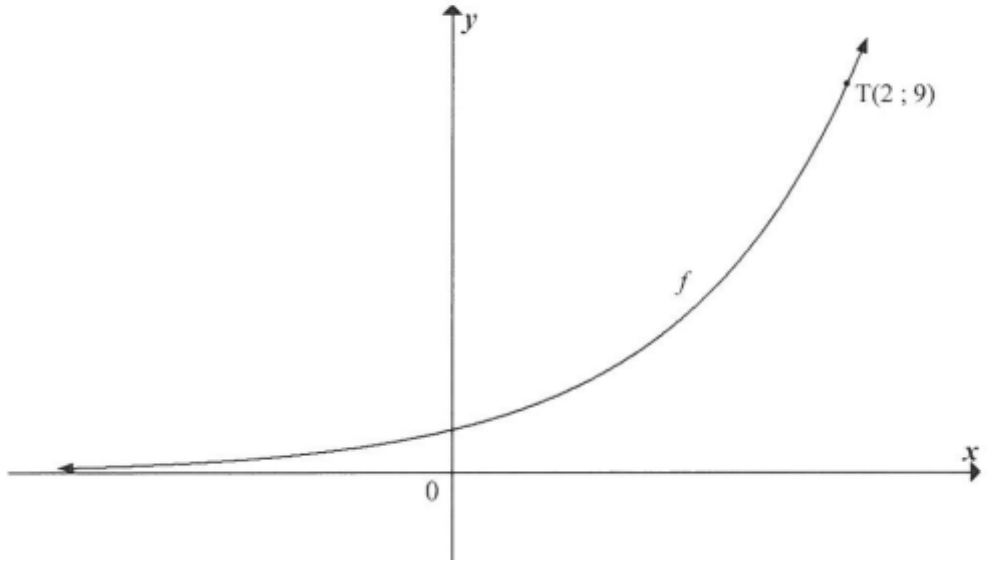
4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$ (3)

4.5 Determine the value(s) of x for which $\frac{6}{x+2} - 1 \geq -x - 3$. (2)
[13]

Question 5 **Feb March 2015**

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$. (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

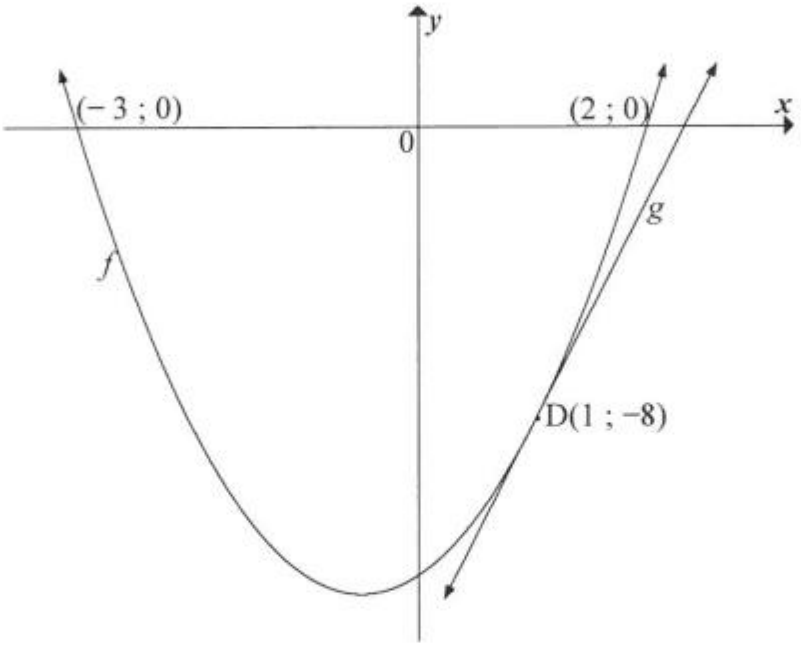
5.4 Is the inverse of f a function? Explain your answer. (2)
[7]

Question 6 **Feb March 2015**

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D(1 ; – 8) is a common point on f and g.

- f intersects the x-axis at (– 3 ; 0) and (2 ; 0).
- g is the tangent to f at D.



6.1 For which value(s) of x is $f(x) \leq 0$? (2)

6.2 Determine the values of a, b and c. (5)

6.3 Determine the coordinates of the turning point of f. (3)

6.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x-7) + 2$. (2)

6.5 Calculate the gradient of g. (3)
[15]

Question 4 November 2015

Given: $f(x) = 2^{x+1} - 8$

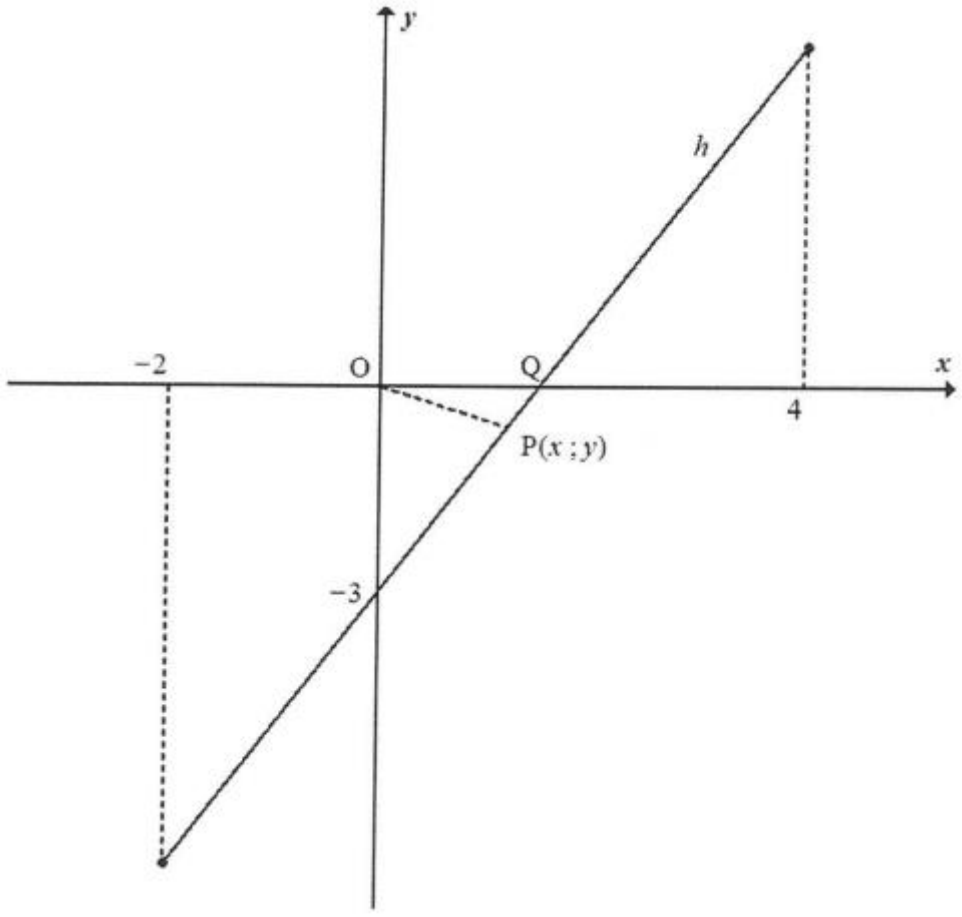
4.1 Write down the equation of the asymptote of f. (1)

4.2 Sketch the graph of f. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g. (1)

[6]

Question 5 November 2015



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 P$(x\ ;\ y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \le x \le 4$.

5.6.1 Explain why f has a local minimum. (2)

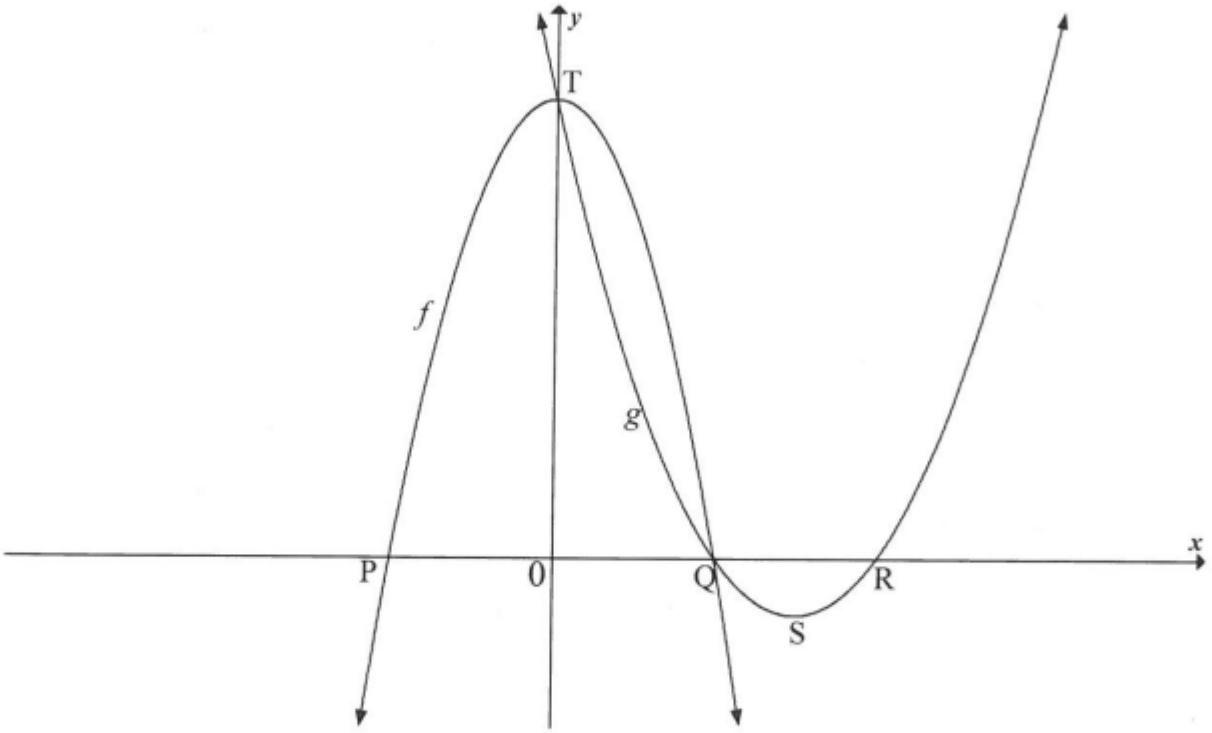
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

[19]

Question 6 **November 2015**

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R,\ x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[12]

Question 4 **Feb March 2016**

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the y-intercept of f. (1)

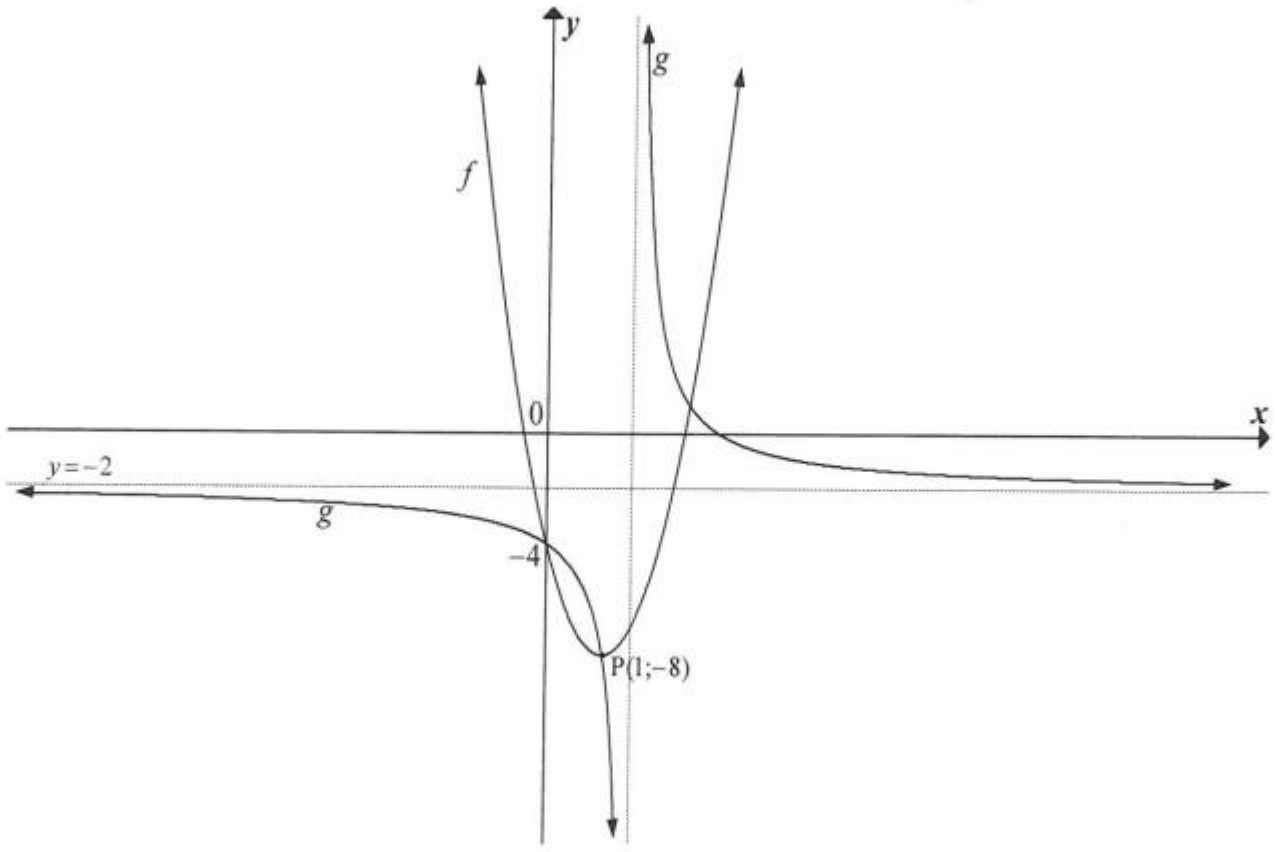
4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$. (3)

4.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h. (1)

[8]

Question 5 **Feb March 2016**



The graphs of the functions $f(x)=a(x+p)^2+q$ and $g(x)=\frac{k}{x+r}+d$ are sketched below.

Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of f. The horizontal asymptote of g is $y=-2$.

5.1 Calculate the values of a, p and q. (4)

5.2 Calculate the values of k, r and d. (6)

5.3 Determine the value(s) of x in the interval $x\leq 1$ for which $g(x)\geq f(x)$. (2)

5.4 Determine the value(s) of k for which $f(x)=k$ has two, unequal positive roots. (2)

5.5 Write down an equation for the axis of symmetry of g that has a negative gradient. (3)

5.6 The point P is reflected in the line determined in QUESTION 5.5 to give the point Q. Write down the coordinates of Q. (2)
[19]

Question 6 **Feb March 2016**

Given: $f(x)=\frac{1}{4}x^2, x\leq 0$

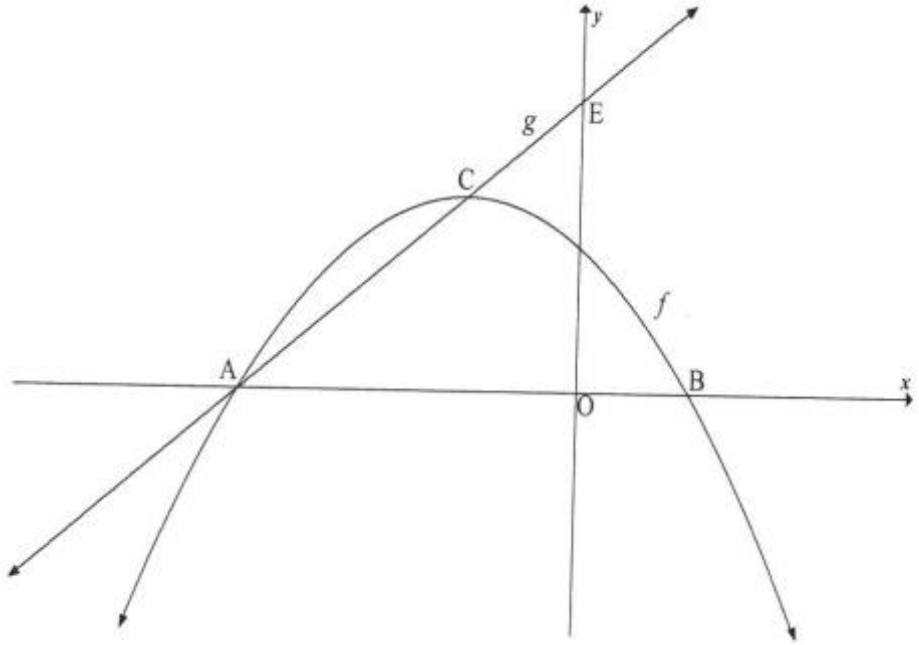
6.1 Determine the equation of f^{-1} in the form $f^{-1}(x)=\ldots$ (3)

6.2 On the same system of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}. (3)

6.3 Is f^{-1} a function? Give a reason for your answer. (2)
[8]

Question 4 **May June 2016**

The sketch below shows the graphs of $f(x)=-x^2-2x+3$ and $g(x)=mx+q$.
Graph f has x-intercepts at A and B(1 ; 0) and a turning point at C.
The straight line g, passing through A and C, cuts the y-axis at E.



4.1 Write down the coordinates of the y-intercept of f. (1)

4.2 Show that the coordinates of C are $(-1;4)$. (3)

4.3 Write down the coordinates of A. (1)

4.4 Calculate the length of CE. (6)

4.5 Determine the value of k if $h(x)=2x+k$ is a tangent to the graph of f. (5)

4.6 Determine the equation of g^{-1}, the inverse of g, in the form $y=\ldots$ (2)

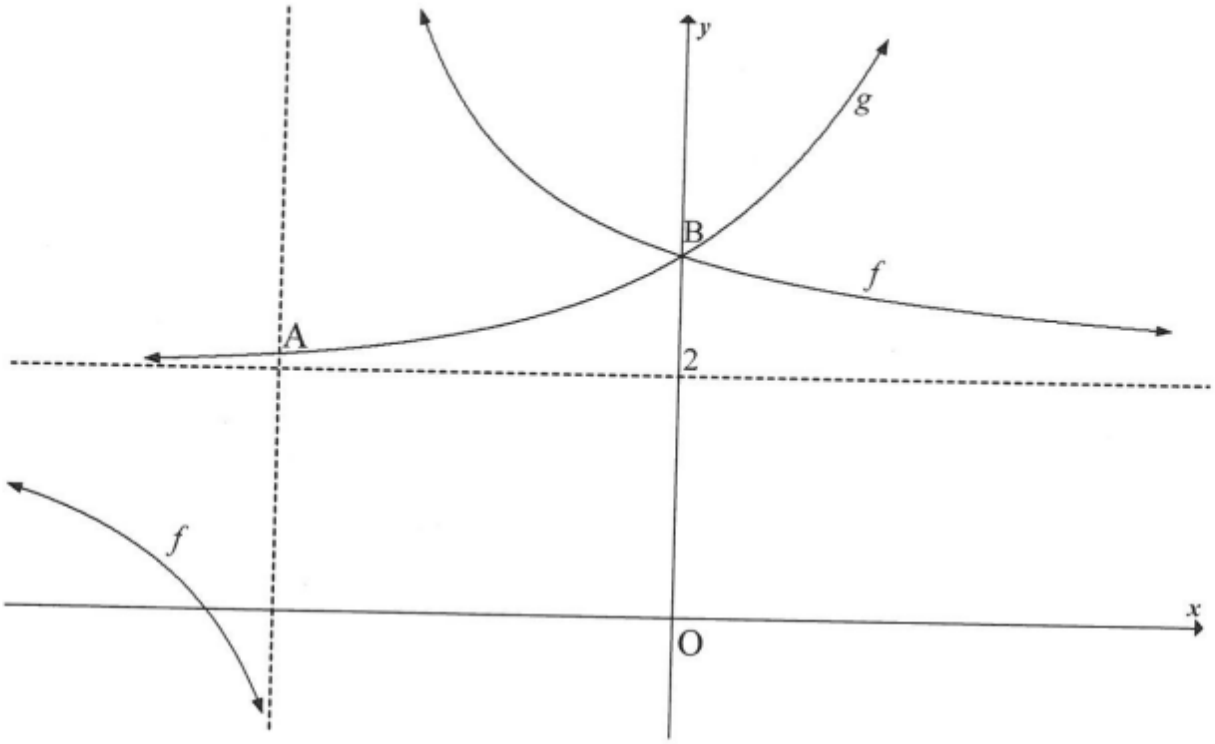
4.7 For which value(s) of x is $g(x)\geq g^{-1}(x)$? (3)

[21]

Question 5 — May June 2016

The sketch below shows the graphs of $f(x)=\dfrac{3}{x-p}+q$ and $g(x)=2^x+r$

- g intersects the vertical asymptote of f at A.
- B is the common y-intercept of f and g.
- $y=2$ is the common horizontal asymptote of f and g.



5.1 Write down the value of r. (1)

5.2 Determine the value of p. (4)

5.3 Determine the coordinates of A. (3)

5.4 For which value(s) of x is $f(x)-g(x)\geq 0$? (2)

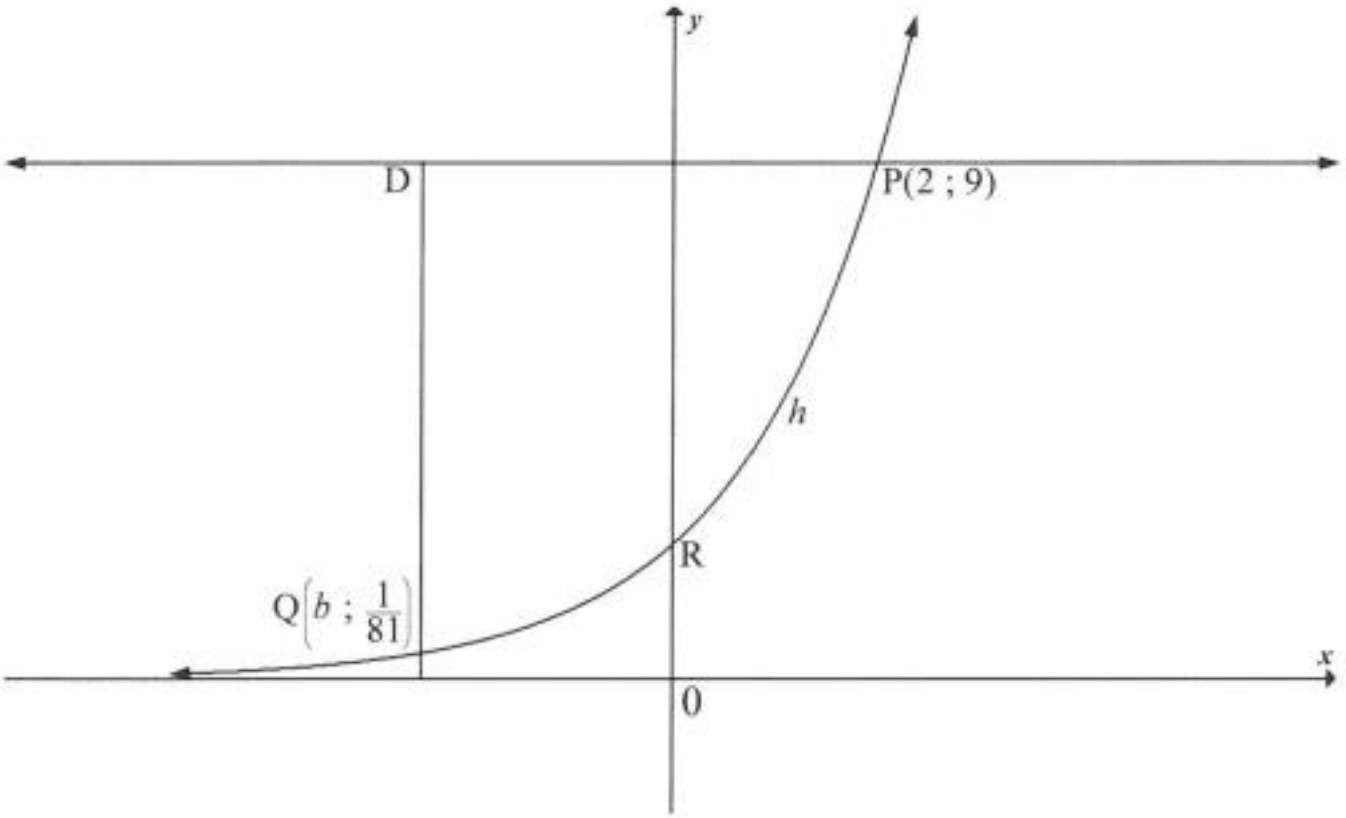
5.5 If $h(x)=f(x-2)$, write down the equation of h. (2)

[12]

Question 4 **November 2016**

Sketched below is the graph of $h(x)=a^x$, $a>0$. R is the y-intercept of h.

The points P(2 ; 9) and $Q\left(b\,;\frac{1}{81}\right)$ lie on h.



4.1 Write down the equation of the asymptote of h. (1)

4.2 Determine the coordinates of R. (1)

4.3 Calculate the value of a. (2)

4.4 D is a point such that DQ $\parallel$ y-axis and DP $\parallel$ x-axis. Calculate the length of DP. (4)

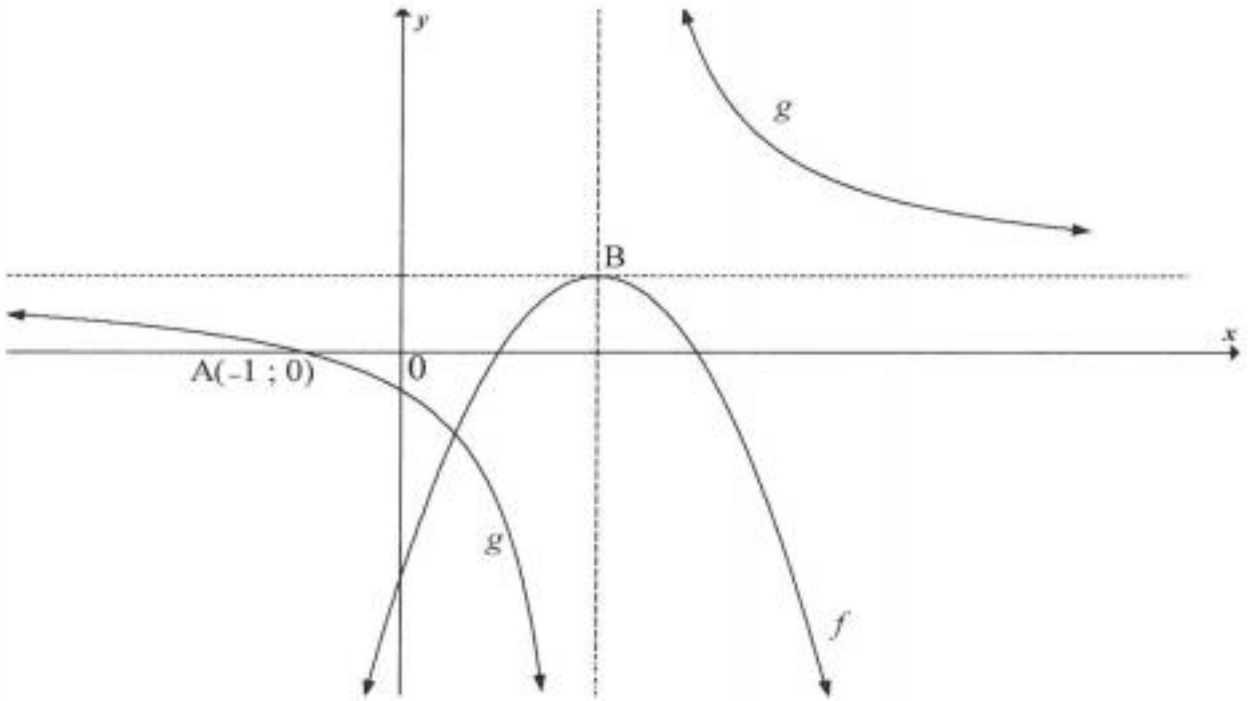
4.5 Determine the values of k for which the equation $h(x+2)+k=0$ will have a root that is less than -6. (3)

[11]

Question 5 **November 2016**

Sketched below is the parabola f, with equation $f(x)=-x^2+4x-3$ and a hyperbola g, with equation $(x-p)(y+t)=3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- A$(-1\,;0)$ is the x-intercept of g.



5.1 Show that the coordinates of B are (2 ; 1) (2)

5.2 Write down the range of f. (1)

5.3 For which value(s) of x will $g(x) \geq 0$? (2)

5.4 Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x+4)$ (1)

5.5 Determine the values of p and t. (4)

5.6 Write down the values of x for which $f(x).g'(x) \geq 0$ (4)

[14]

Question 6 **November 2016**

Given: $f(x) = -x + 3$ and $g(x) = \log_2 x$

6.1 On the same set of axes, sketch the graphs of f and g, clearly showing ALL intercepts with the axes. (4)

6.2 Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$ (2)

6.3 Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3-x) = x$. (3)

6.4 Write down the solution to $\log_2(3-x) = x$. (1)

[10]

Question 7 November 2014

7.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation. (3)

7.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

7.2.1 Calculate the value of Samuel's monthly instalment. (4)

7.2.2 Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

7.2.3 Who pays more interest, Samuel or Melissa? Justify your answer. (2)
[13]

Question 7 Feb March 2015

7.1 Nomsa started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into an annuity fund. She continued doing this until she retired on 31 December 2013.

7.1.1 Determine the total amount of money that she paid into the fund. (2)

7.1.2 The interest rate on this fund was 8% p.a., compounded monthly. Calculate the value of the fund at the time that she retired. (5)

7.1.3 On 1 January 2014 Nomsa invested R2 million in an account paying interest at 10% p.a. compounded monthly. Nomsa withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If Nomsa wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount she could withdraw at the end of each month. (4)

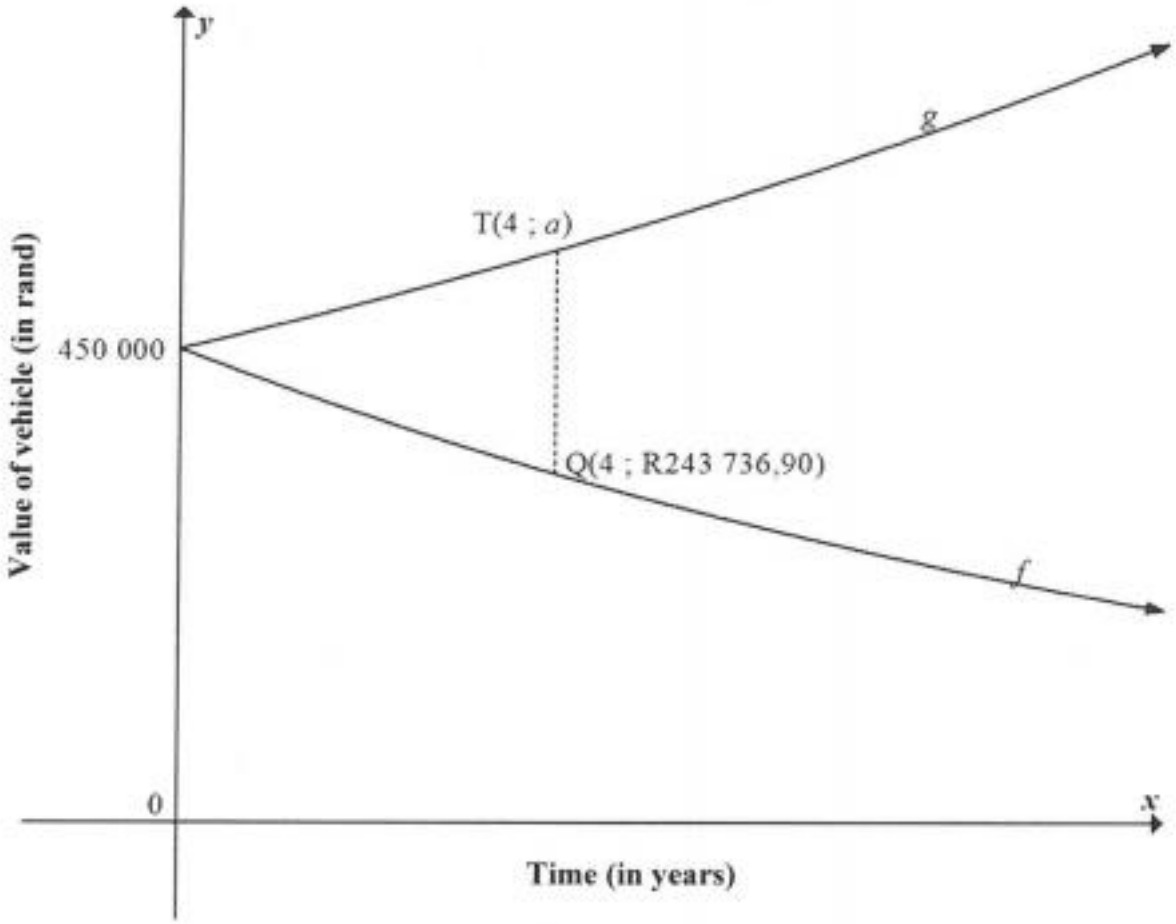
7.2 For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.

At the end of 2012 the populations of these two towns were equal.

Determine the ratio of the population of town X (call it P_X) to the population of town Y (call it P_Y) at the beginning of 2010. (4)
[15]

Question 7 **November 2015**

The graph of f shows the book value of a vehicle x years after the time Joe bought it.
The graph of g shows the cost price of a similar new vehicle x years later.



7.1 How much did Joe pay for the vehicle? (1)

7.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)

7.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3)

7.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund. (5)
[13]

Question 7 **Feb March 2015**

7.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.

7.1.1 What is the quarterly interest rate for Diane's investment? (1)

7.1.2 Calculate the amount in Diane's savings account at the end of the 2 years. (3)

7.2 Motloi inherits R800 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.

7.2.1 How many withdrawals of R10 000 will Motloi be able to make from this fund? (5)

7.2.2 Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and to use it as a deposit towards a house.

(a) What is the value of Motloi's deposit, to the nearest rand? (4)

(b) Motloi's deposit is exactly 30% of the purchase price of the house. What is the purchase price of the house, to the nearest rand? (1)

[14]

Question 6 **May June 2016**

6.1 How long would the price of an asset take to reduce by a third of its original value if it depreciates on a reducing balance at a rate of 4,7% p.a.? (4)

6.2 Lebogo bought a tractor for Rx on 1 April 2016.

- She will trade in this tractor when she replaces it with a similar one in 5 years' time on 1 April 2021.
- The tractor depreciates by 20% p.a. according to the reducing-balance method.
- The price of a similar tractor increases by 18% annually.
- Lebogo calculated that if she deposited R8 000 per month into a sinking fund, which paid interest at 10% p.a. compounded monthly, she would have enough money to cover the replacement cost of the tractor. She made the first deposit in this fund on 30 April 2016 and will continue to do so at the end of every month until 31 March 2021.

6.2.1 Determine, in terms of x, what the book value of the current tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to FIVE decimal places. (2)

6.2.2 Determine, in terms of x, what the price of a similar new tractor will be on 1 April 2021. Give your answer correct to FIVE decimal places. (2)

6.2.3 Calculate the amount accumulated in the sinking fund on 1 April 2021. (4)

6.2.4 Calculate the value of x, the price of the current tractor. Round off your answer to the nearest thousand. (4)

[16]

Question 7 **November 2016**

On 1 June 2016 a bank granted Thabiso a loan of R250 000 at an interest rate of 15% p.a. compounded monthly, to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2016 and ending 4 years later on 1 June 2020. However, Thabiso was unable to make the first two instalments and only commenced with the monthly instalments on 1 September 2016.

7.1 Calculate the amount Thabiso owed the bank on 1 August 2016, a month before he paid his first monthly instalment. (2)

7.2 Having paid the first monthly instalment on 1 September 2016, Thabiso will still pay his last monthly instalment on 1 June 2020. Calculate his monthly instalment. (4)

7.3 If Thabiso paid R9 000 as his monthly instalment starting on 1 September 2016, how many months sooner will he repay the loan? (5)

7.4 If Thabiso paid R9 000 as a monthly instalment starting on 1 September 2016, calculate the final instalment to repay the loan. (4)

[15]

Question 8 **November 2014**

8.1 Determine $f'(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)

[14]

Question 9 **November 2014**

Given: $f(x) = (x+2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

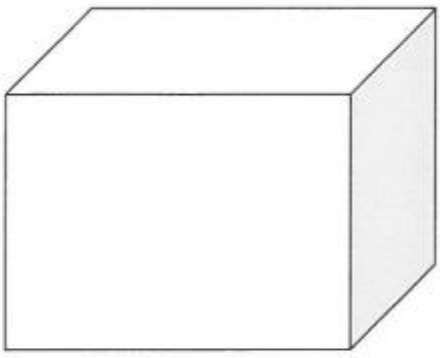
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

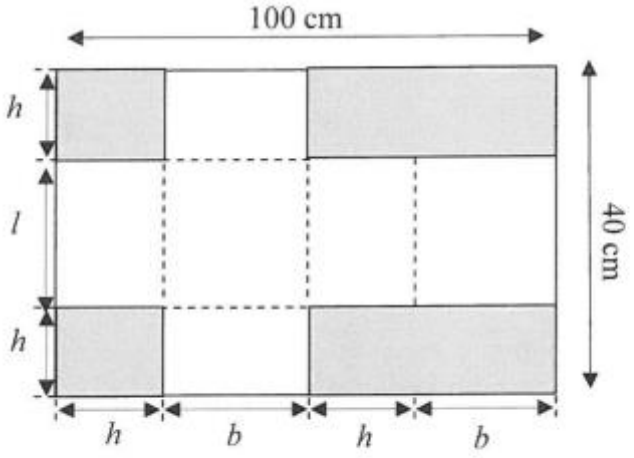
9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f'(x) < 0$? (3)

[13]

Question 10 **November 2014**





A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)

[9]

Question 8 — Feb March 2015

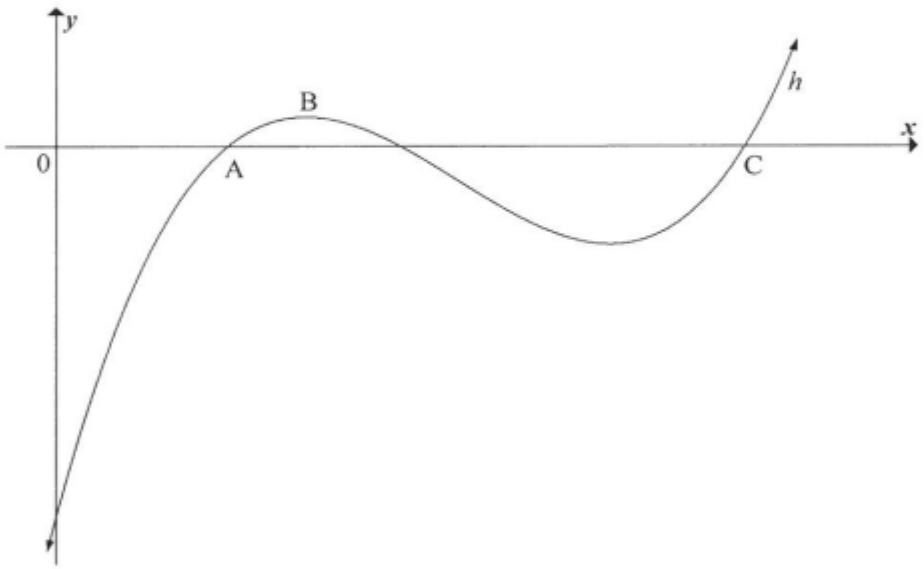
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

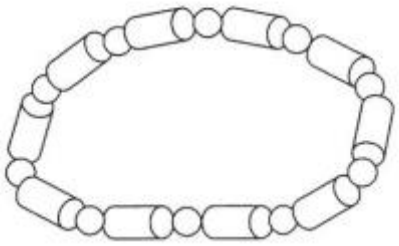
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)

[22]

Question 9 — Feb March 2015

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

9.1 If the volume of a cylinder is $6\ cm^3$, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)

9.3 Determine the value of r so that the least amount of paint will be used. (4)
[9]

Question 8 **November 2015**

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)
[11]

Question 9 **November 2015**

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

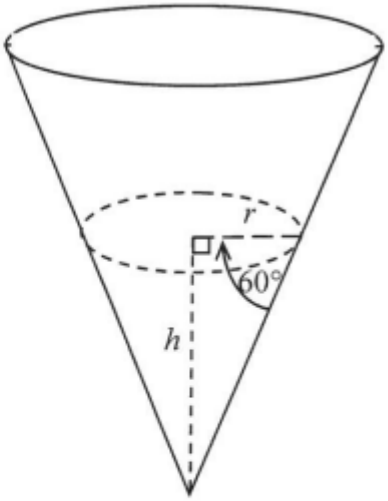
9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)
[17]

Question 10 **November 2015**

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

Question 8 **Feb March 2016**

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

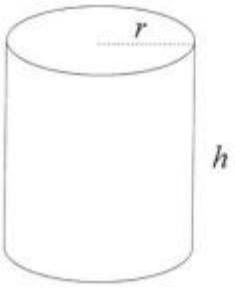
8.3.3 Determine the x-coordinates of the turning points of f. (4)

8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

Question 9 **Feb March 2016**



A soft drink can has a volume of $340\ cm^3$, a height of h cm and a radius of r cm.

9.1 Express h in terms of r. (2)

9.2 Show that the surface area of the can is given by $A(r)=2\pi r^2+680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)
[8]

Question 7 **May June 2016**

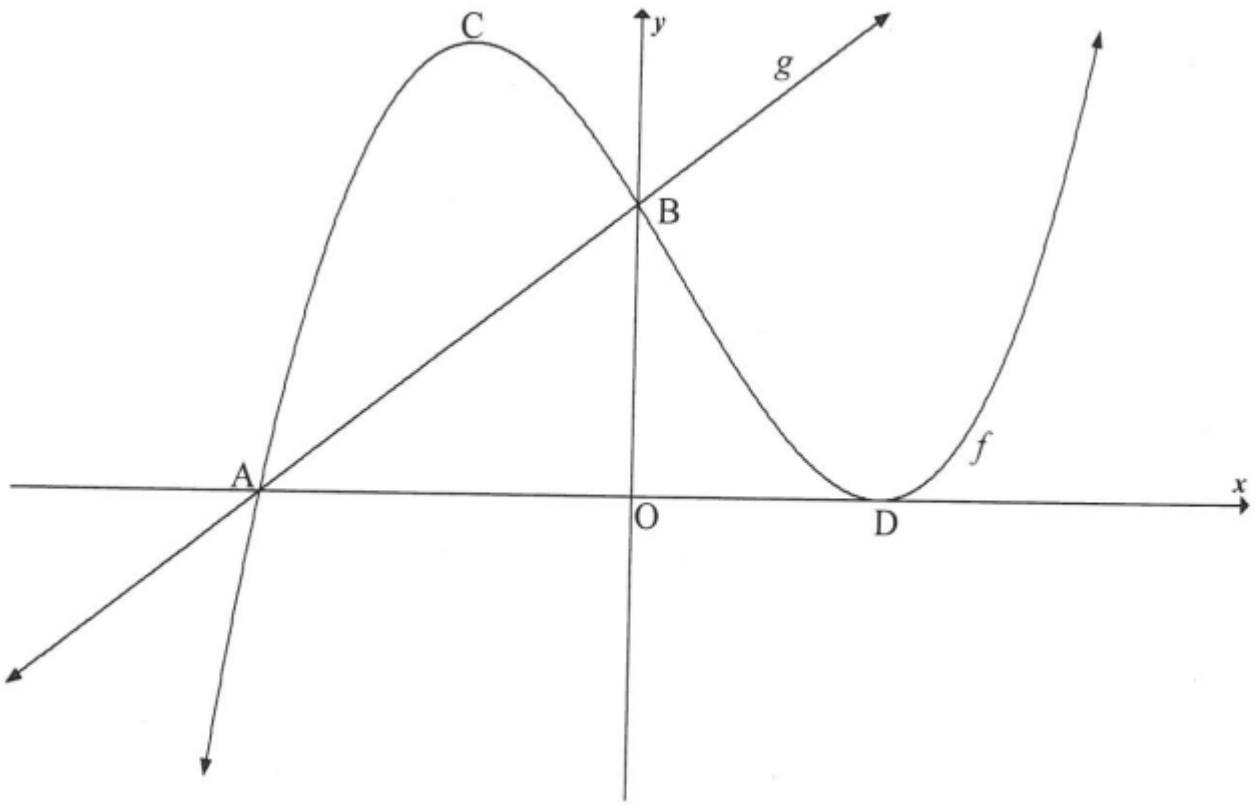
7.1 Determine $f'(x)$ from first principles if $f(x)=3x^2-5$ (5)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y=2x^5+\frac{4}{x^3}$ (3)

7.2.2 $y=\left(\sqrt{x}-x^2\right)^2$ (4)
[12]

Question 8 **May June 2016**



Sketched below are the graphs of $f(x)=(x-2)^2(x-k)$ and $g(x)=mx+12$

- A and D are the x-intercepts of f.
- B is the common y-intercept of f and g.
- C and D are turning points of f.
- The straight line g passes through A.

8.1 Write down the y-coordinate of B. (1)

8.2 Calculate the x-coordinate of A. (3)

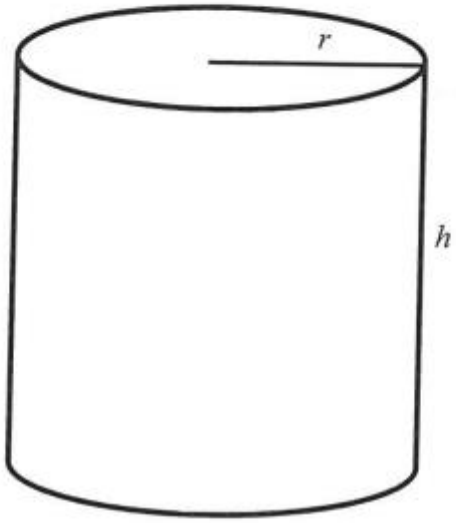
8.3 If $k=-3$, calculate the coordinates of C. (6)

8.4 For which values of x will f be concave down? (3)

[13]

Question 9 **May June 2016**

A 340 mℓ can with height h cm and radius r cm is shown below.



$1\ \text{m}\ell = 1\ \text{cm}^3$

9.1 Determine the height of the can in terms of the radius r. (3)

9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

[9]

Question 8 November 2016

8.1 Determine $f'(x)$ from first principles if $f(x)=3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x=a$, and arrives at the answer: $\lim\limits_{h\to 0}\dfrac{\sqrt{4+h}-2}{h}$

Write down the equation of g and the value of a. (2)

8.3 Determine $\dfrac{dy}{dx}$ if $y=\sqrt{x^3}-\dfrac{5}{x^3}$ (4)

8.4 $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x=1$. Calculate the values of a and b. (5)

[16]

Question 9 November 2016

For a certain function f, the first derivative is given as $f'(x)=3x^2+8x-3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x)=ax^3+bx^2+cx+d$ and $f(0)=-18$, determine the equation of f. (5)

[13]

Question 10 November 2016

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t)=-t^3+3t^2+72t$, $0<t<10$.

10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)

[8]

Question 11 **November 2014**

A survey concerning their holiday preferences was done with 180 staff members. The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

11.1 Determine the probability that a randomly selected staff member:

11.1.1 Is male (1)

11.1.2 Does not prefer visiting a game park (2)

11.2 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)
[7]

Question 12 **November 2014**

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 All the letters of the alphabet can be used (2)

12.1.2 The password must start with a 'D' and end with an 'L' (2)

12.2 Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

12.2.1 In how many different ways can ALL the cars be parked? (2)

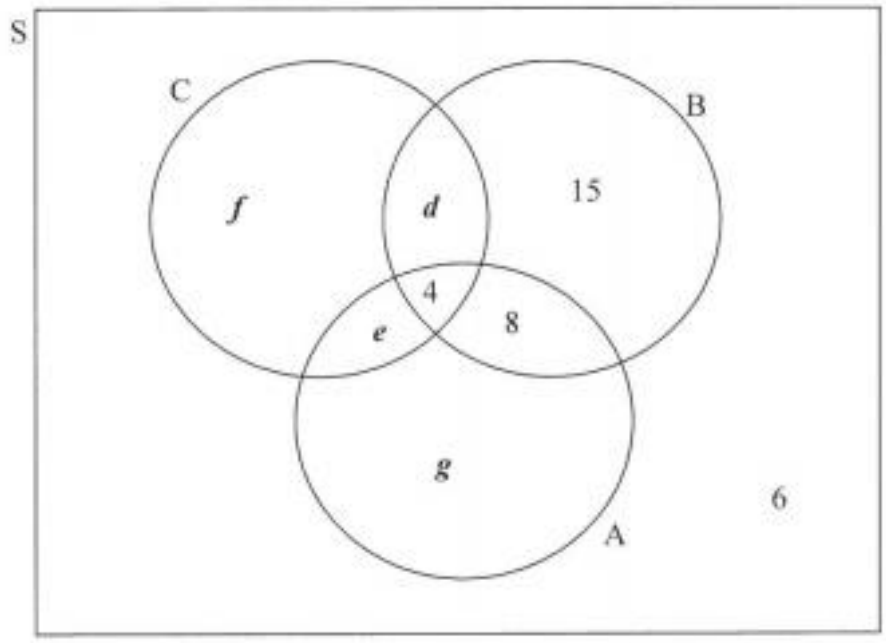
12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)
[9]

Question 10 **Feb March 2015**

10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



10.1.1 Use the given information and the Venn diagram to determine the values of d, e, f and g. (4)

10.1.2 For a randomly selected country, calculate:

(a) P(A and B and C) (1)

(b) P(A or B or C) (1)

(c) P(only C) (1)

(d) P(that a country uses exactly two methods) (1)

10.2 Nametso may choose DVDs from three categories as listed in the table below:

Drama	Romance	Comedy
• *Last Hero*	• *One Heart*	• *Laughing Dragon*
• *Midnight*	• *You and Me*	• *Falling Down*
• *Stranger Calls*	• *Love Song*	• *Sitting on the Stairs*
• *Missing in Action*	• *Bird's First Nest*	
• *Only 40 Seconds Left*		

10.2.1 Nametso must choose ONE DVD from the Drama category. What is the probability that she will choose *Midnight*? (2)

10.2.2 How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy? (2)

10.2.3 Calculate the probability that she will have *Last Hero* and *Laughing Dragon* as part of her selection in QUESTION 10.2.2. (2)

[14]

Question 11 **November 2015**

11.1 For two events, A and B, it is given that:

$P(A) = 0,2$
$P(B) = 0,63$
$P(A \text{ and } B) = 0,126$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)

[17]

Question 10 **Feb March 2016**

10.1 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	***b***	***c***	80
Fruit juice	***d***	***e***	20
TOTAL	60	100	***a***

10.1.1 Write down the value of ***a***. (1)

10.1.2 What is the probability that a randomly selected passenger is male? (2)

10.1.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of ***b***. (4)

10.2 A Banana Airways aeroplane has 6 seats in each row.

10.2.1 How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)

10.2.2 Xoliswa, Anees and 4 other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must sit next to each other? (2)

10.2.3 Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? (4)

[15]

Question 10 **May June 2016**

10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

10.1.1 What is the probability that a member selected at random is:

(a) Female (2)

(b) Female and plays tennis (1)

10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

10.2 The probability of events A and B occurring are denoted by P(A) and P(B) respectively.

For any two events A and B it is given that:

- $P(B') = 0,28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0,96$

Are events A and B mutually exclusive? Justify your answer. (4)
[10]

Question 11 **May June 2016**

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)

11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)
[6]

Question 11 **November 2016**

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Male	80	*a*	
Female	48	12	
Totals	*b*	32	160

11.1 Calculate the values of *a* and *b*. (2)

11.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)

11.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:

11.3.1 Watched TV in the period during which the examinations were written? (2)

11.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2)
[8]

Question 12 **November 2016**

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? **[5]**